

# **Cooperative Game Theory for Cognitive Radio**



Zhu Han

Department of Electrical and Computer Engineering  
University of Houston, Houston, TX, USA

Gamecomm, 5/16/11

Thanks for US NSF Career Award and Dr. Walid Saad

# Outline

---

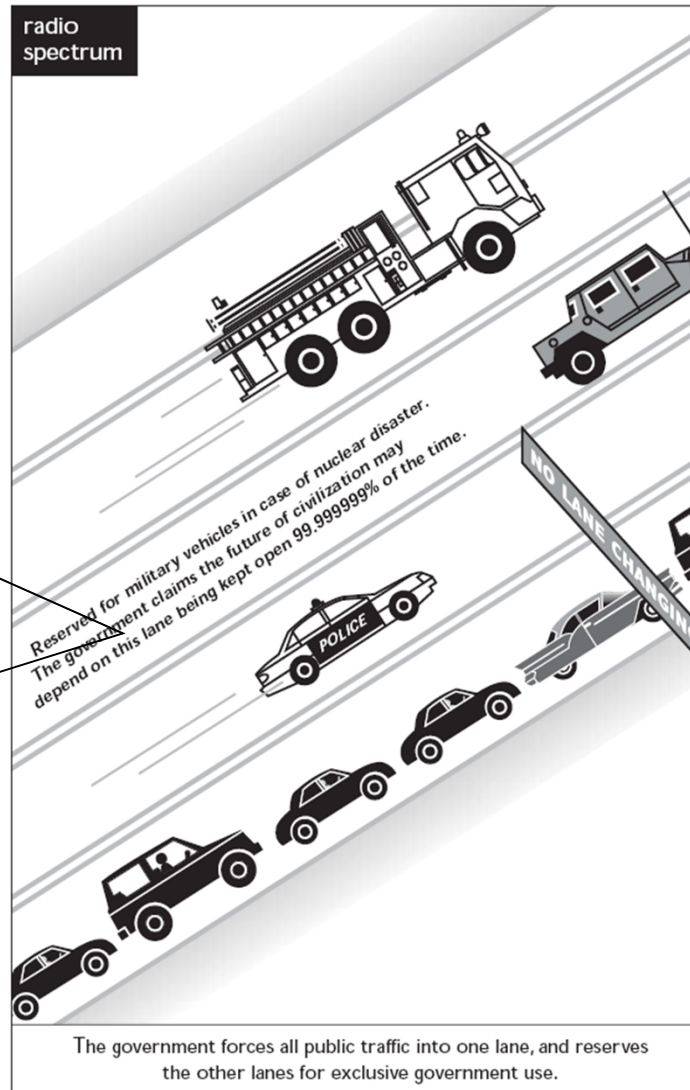
- Cognitive Radio Networks
  - Spectrum Sensing
  - Dynamic Spectrum Access
  - Exploration and Exploitation
- Overview of Game Theory
- Coalitional Games
  - Class I: Canonical Coalitional Games
  - Class II: Coalition Formation Games
  - Class III: Coalition Graph Games
- Conclusions



# Cognitive Radio Overview

Lane: spectrum  
Car: mobile user

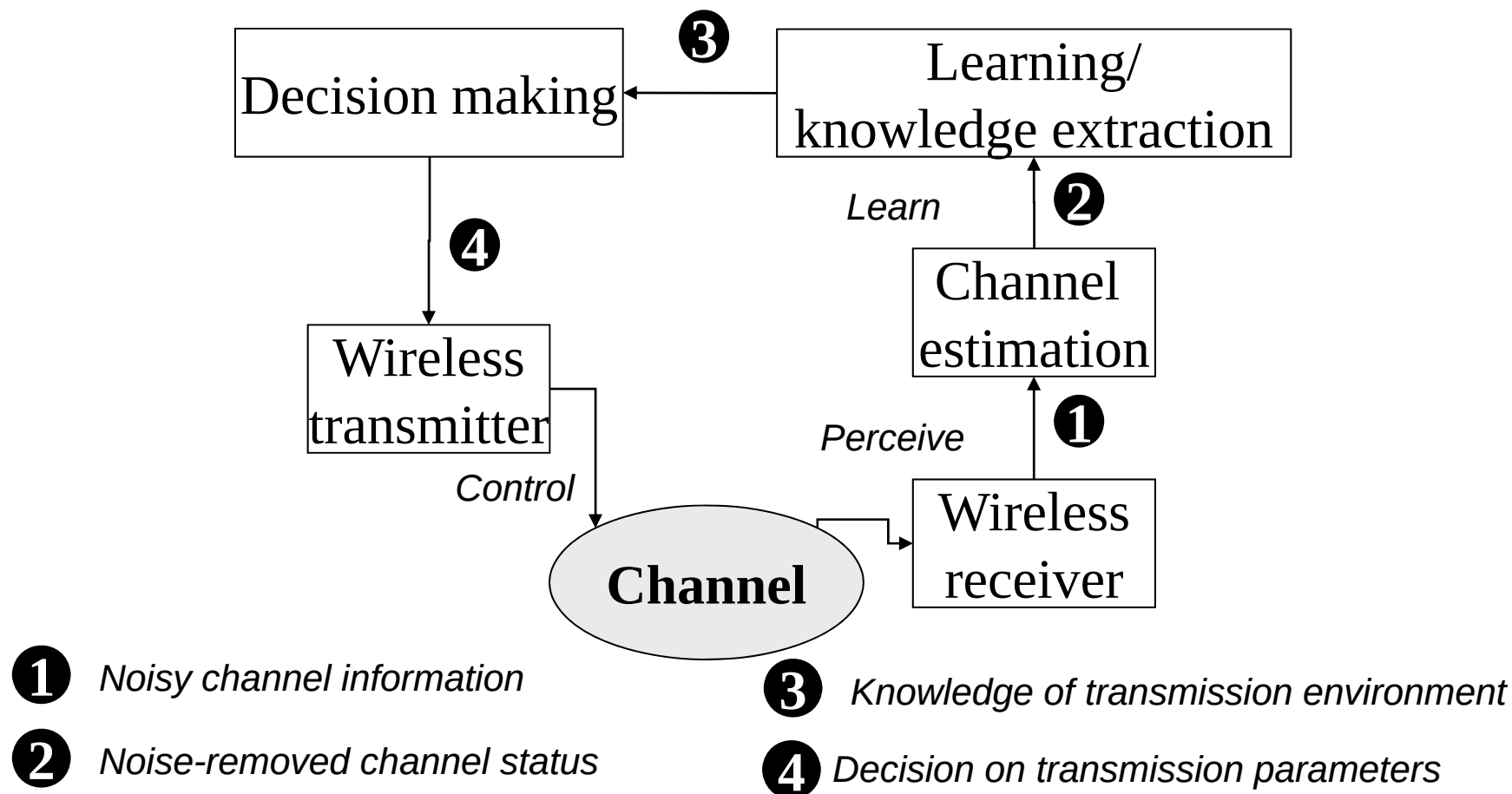
Lane seldom  
used but  
reserved for  
**Licensed  
Spectrum  
Or  
Primary  
Users**



Public Traffic  
Lane congested  
for  
**Unlicensed  
Spectrum  
Or  
Secondary Users**

# Cognitive Radio Block Diagram

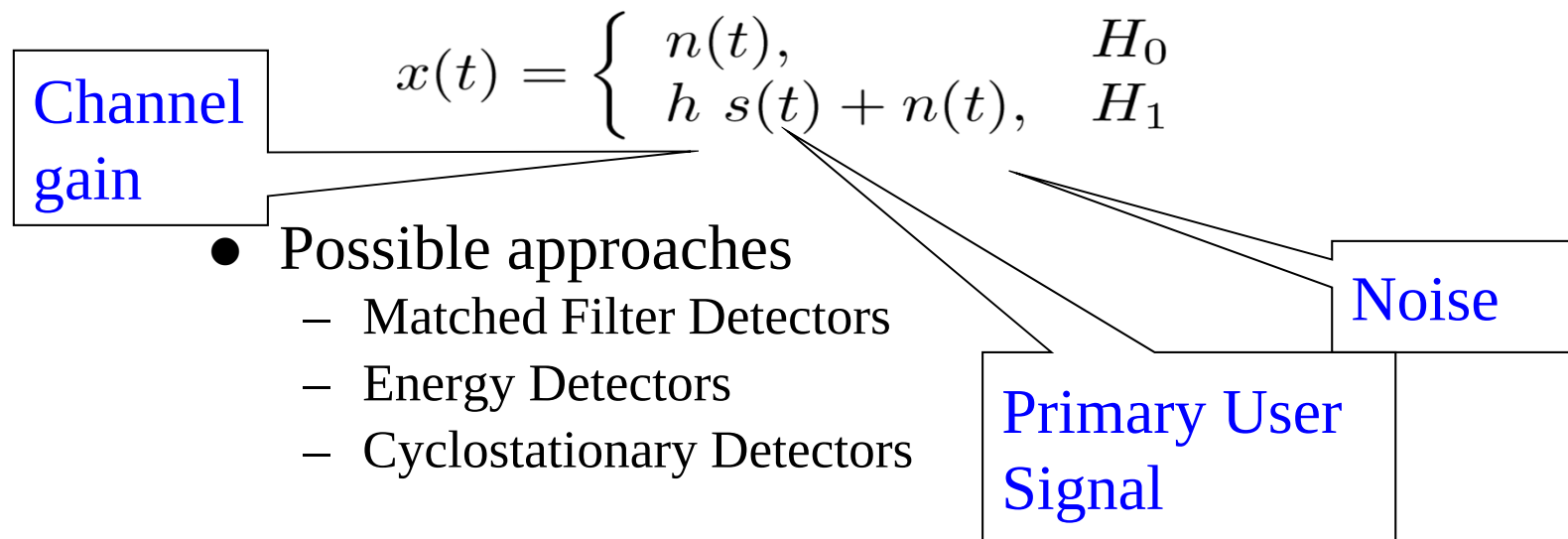
- A cognitive radio is a radio that is able to sense, adapt and learn from its operating environment



# Problem 1: Spectrum Sensing

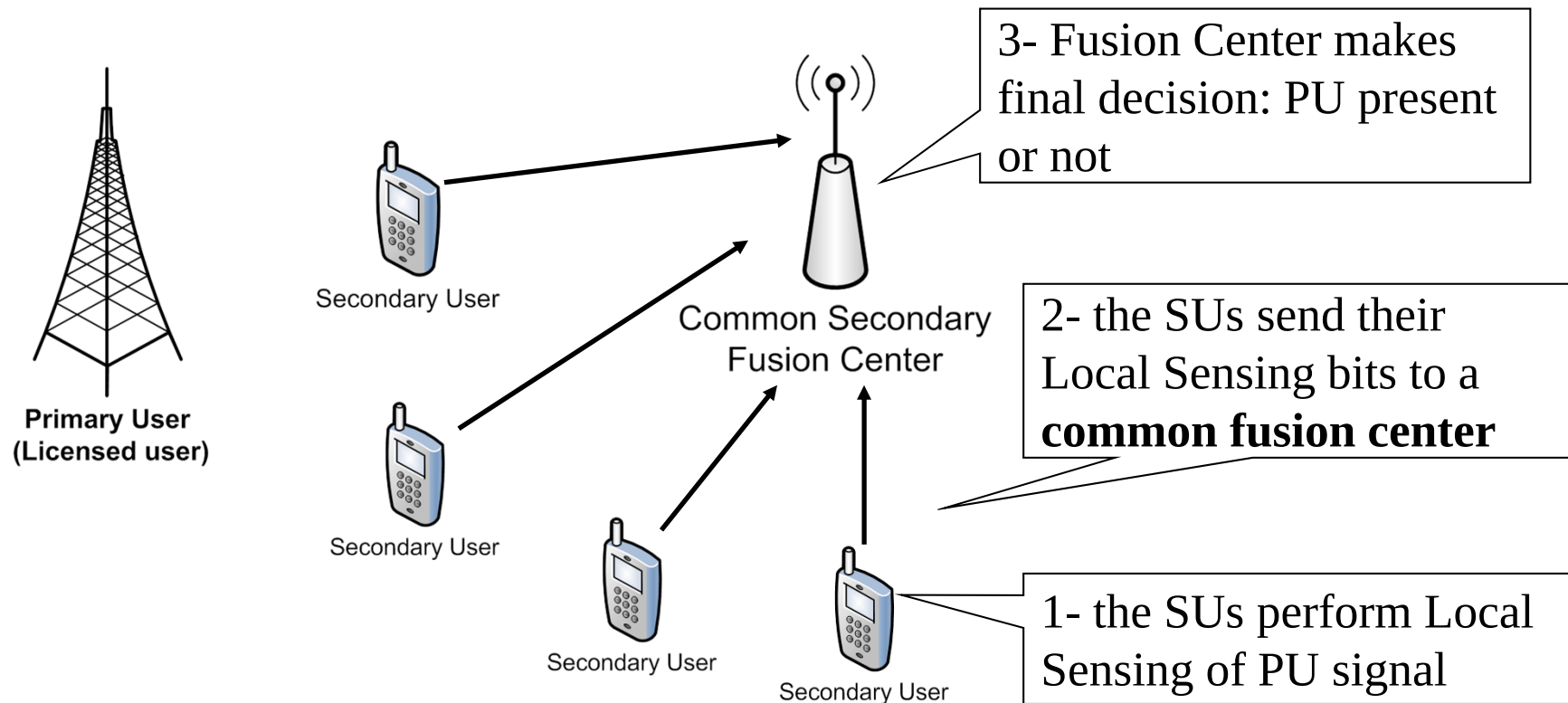
---

- Secondary users must sense the spectrum to
  - Detect the presence of the primary user for reducing interference on primary user
  - Detect spectrum holes to be used for dynamic spectrum access
- Spectrum sensing is to make a decision between two hypotheses
  - The primary user is present, hypothesis  $H_1$
  - The primary user is absent, hypothesis  $H_0$



# Collaborative Spectrum Sensing

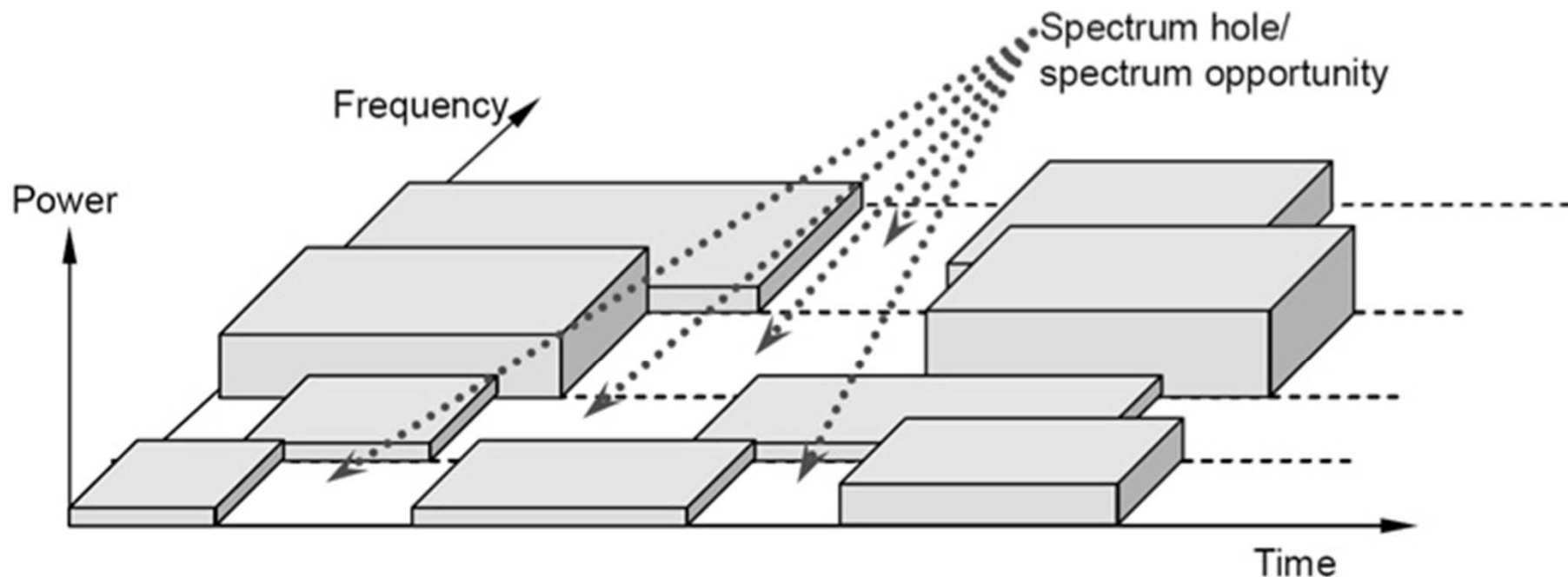
- Overcome hidden terminal problem
- Multiple cognitive radio observe together



## ***Problem 2: Dynamic Spectrum Access***

---

- Adjust spectrum resource usage in the near-real-time manner in response to changes in the users' objectives, changes of radio states, and changes in the environment and external constraints.

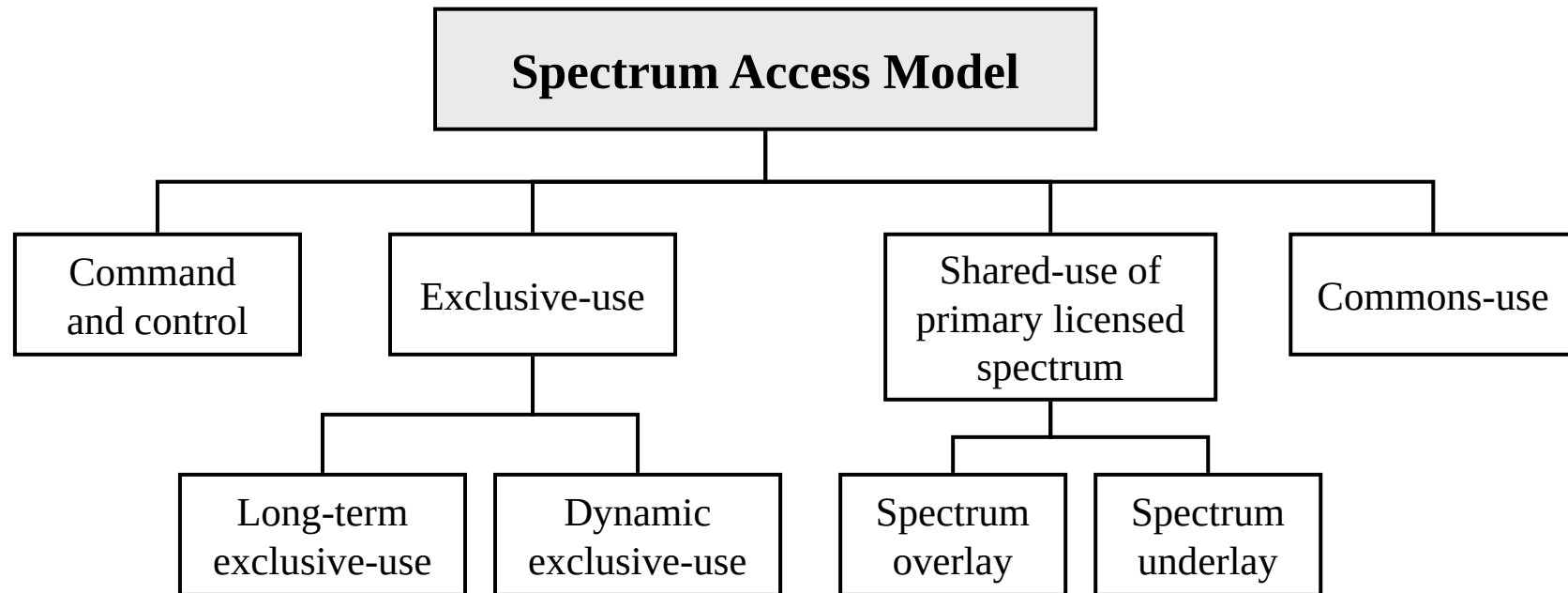


**Access channels by multiple secondary users: Random vs. Coordinate; non-cooperative vs. cooperative; social optimum vs. fairness, ....**

# *Dynamic Spectrum Access (DSA)*

---

- Dynamic spectrum access allows different wireless users and different types of services to utilize radio spectrum





## ***Problem 3: Exploration and Exploitation***

- Exploitation: the immediate benefit gained from accessing the channel with the estimated highest reward
- Exploration is the process by which the cognitive users tend to probe more channels to discover better channel opportunities.
- Example: should find new topics or study the current topics



Access existing  
spectrum holes

Search for new  
spectrum holes



# Game Theory Overview

---

- What is game theory?
  - The formal study of conflict or cooperation
  - Modeling mutual interaction among rational decision makers
  - Widely used in economics
- Components of a “game”
  - **Rational players** with conflicting interests or mutual benefit
  - **Strategies** or actions
  - **Utility** as a payoff of player's and other players' actions
  - **Outcome**
- Many types
  - Non-cooperative game theory
  - Cooperative game theory
  - Dynamic game theory
  - Stochastic game
  - Auction theory



# Rich Game Theoretical Approaches

- Non-cooperative static

game: play once

Prisoner Dilemma

Payoff: (user1, user2)

- Mandayam and Goodman (2001)
- Virginia tech

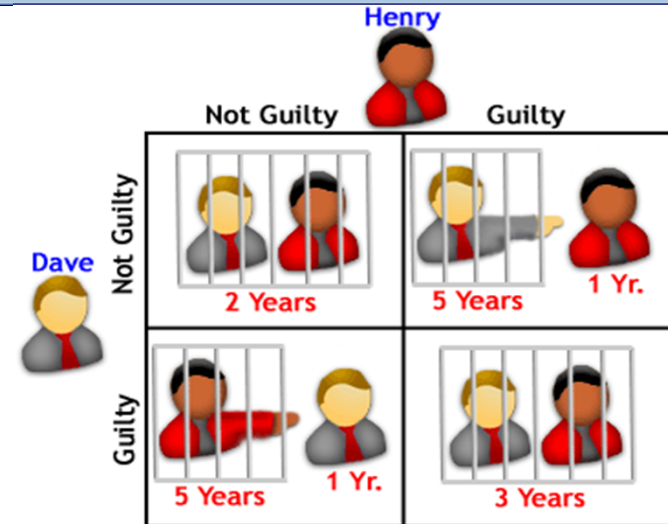
- Repeated game: play multiple times

- **Threat of punishment** by repeated game. MAD: Nobel prize 2005.
- Tit-for-Tat (infocom 2003):

- Dynamic game: (Basar's book)

- ODE for state
- Optimization utility over time
- HJB and dynamic programming
- Evolutional game (Hossain and Dusit's work)

- Stochastic game (Altman's work)



A 2x2 matrix illustrating the Prisoner Dilemma. The columns are labeled 'Not Guilty' and 'Guilty' with a character icon above them. The rows are labeled 'Not Guilty' and 'Guilty' with a character icon to the left. Each cell shows two characters in a prison cell with their respective prison terms below them.

|            | Not Guilty     | Guilty           |
|------------|----------------|------------------|
| Not Guilty | 2 Years, 1 Yr. | 5 Years, 1 Yr.   |
| Guilty     | 5 Years, 1 Yr. | 3 Years, 3 Years |

Copyright 2005 - Investopedia.com

# *Auction Theory*

---

Book of Myerson (Nobel Prize 2007), J. Huang, H. Zheng, X. Li



# Cooperative Game Theory

---

- Players have **mutual benefit** to cooperate
  - Startup company: everybody wants IPO, while competing for more stock shares.
  - Coalition in Parlement
- Namely two types
  - Nash bargaining problems
  - Coalitional game
- We will focus on coalitional game theory
  - Definition and key concepts
  - New classification
  - Applications in wireless networks

Walid Saad, Zhu Han, Merouane Debbah, Are Hjørungnes, and Tamer Basar,  
"Coalitional Game Theory for Communication Networks", IEEE Signal  
Processing Magazine, Special Issue on Game Theory, p.p. 77-97, September 2009.



# Coalitional Games: Preliminaries

---

- **Definition** of a coalitional game  $(N, v)$ 
  - A set of players  $N$ , **a coalition**  $S$  is a group of cooperating players (subset of  $N$ )
  - Worth (utility) of a coalition  $v$ 
    - ⌞ In general, **payoff**  $v(S)$  is a real number that represents the gain resulting from a coalition  $S$  in the game  $(N, v)$
    - ⌞  $v(N)$  is the worth of forming the coalition of all users, known as the **grand coalition**
  - User payoff  $x_i$ : the portion of  $v(S)$  received by a player  $i$  in coalition  $S$



# Coalitional Games: Utility

---

- **Transferable utility (TU)**
  - The worth  $v(S)$  of a coalition  $S$  can be distributed arbitrarily among the players in a coalition hence,
  - $v(S)$  is a **function** from the power set of  $N$  over the real line
- **Non-transferable utility (NTU)**
  - The payoff that a user receives in a coalition is pre-determined, and hence the value of a coalition cannot be described by a function
  - $v(S)$  is a set of payoff vectors that the players in  $S$  can achieve
$$v(S) \subseteq \mathbb{R}^{|S|}$$
  - Developed by Auman and Peleg (1960) using a non-cooperative game in strategic form as a basis



# *Payoff division*

---

- Equal fair
  - Each user guarantees its non-cooperative utility
  - The extra worth is divided equally among coalition users
- Proportional fair
  - Each user guarantees its non-cooperative utility
  - A proportional fair division, based on the non-cooperative worth, is done on the extra utility available through cooperation
- Other fairness
  - Shapley value
  - Nucleolus
  - Market Fairness





## ***An example coalitional game***

---

- Example of a coalition game: Majority Vote

$$v(S) = \begin{cases} 1, & \text{if } |S| > |N|/2; \\ 0, & \text{otherwise.} \end{cases}$$

- President is elected by majority vote
- A coalition consisting of a majority of players has a worth of 1 since it is a decision maker
- Value of a coalition does not depend on the external strategies of the users
  - ⌞ *This game is in characteristic function form*
- If the voters divide the value as money
  - ⌞ *Transferable utility*



# Outline

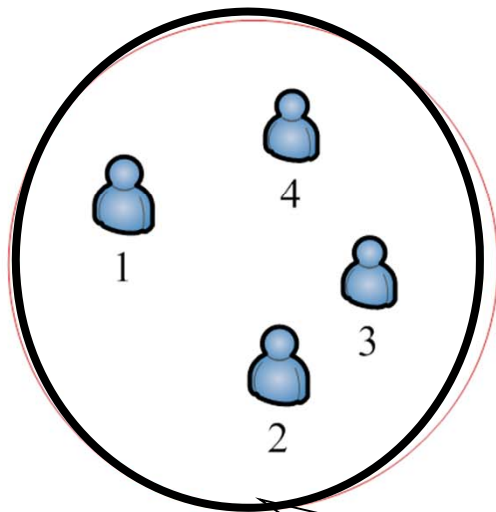
---

- Cognitive Radio Networks
  - Spectrum Sensing
  - Dynamic Spectrum Access
  - Exploration and Exploitation
- Overview of Game Theory
- Coalitional Games
  - Class I: Canonical Coalitional Games
  - Class II: Coalition Formation Games
  - Class III: Coalition Graph Games
- Conclusions

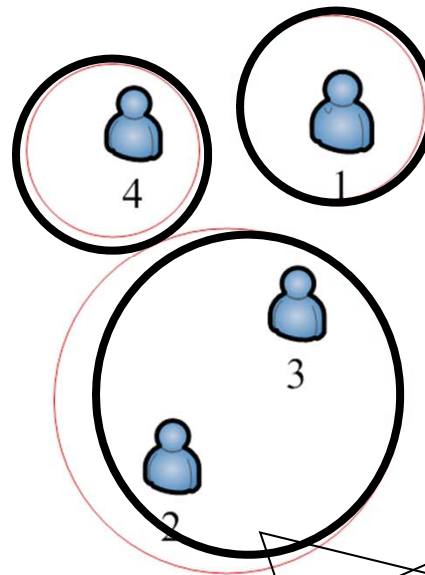


# A new classification

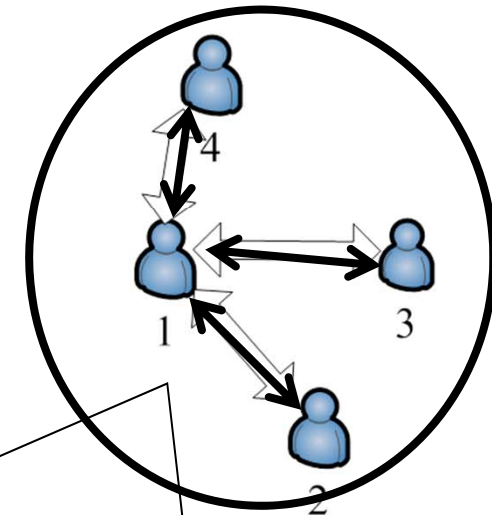
**Class I: Canonical Coalitional Games**



**Class II: Coalition Formation Games**



**Class III: Coalitional Graph Games**



- Players' interactions are governed by a communication graph structure.
- The network structure that forms depends on gains and costs from cooperation.
- Key question: "How to stabilize the grand coalition or form a network structure?"
- Key question: "The grand coalition of all users is an optimal structure taking into account the communication graph!"
- Key question: "How to stabilize the grand coalition?"
- Solutions are complex, combine concepts from coalitions, and non-cooperative games.
- More complex. Several well-defined solution concepts exist.
- Cooperative games

# Class I: Canonical Coalitional Games

---

- Main properties
  - Cooperation is **always** beneficial
    - u *The **grand coalition** is guaranteed to form*
  - The game is **superadditive**
    - $$v(S_1 \cup S_2) \geq v(S_1) + v(S_2) \quad \forall S_1, S_2 \subset N \text{ with } S_1 \cap S_2 = \emptyset$$
  - **The most famous type of coalitional games!**
- Main Objectives
  - Study the properties and **stability** of the grand coalition
    - u *How can we stabilize the grand coalition?*
  - How to divide the utility and gains in a fair manner ?
    - u *Improper payoff division => incentive for players to leave coalition*



# Canonical games: Solution concepts

---

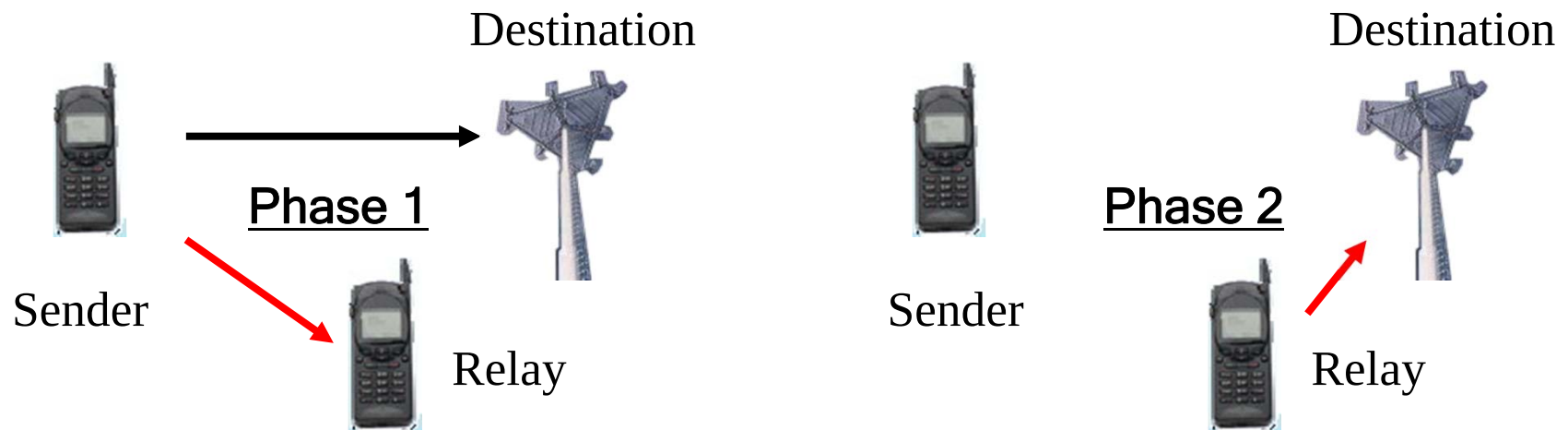
- The **Core**: the most **renowned** concept
  - For a TU game, the core is a set of payoff allocation  $(x_1, \dots, x_N)$  satisfying two conditions
    1.  $\sum_{i \in N} x_i = v(N)$
    2.  $\sum_{i \in S} x_i \geq v(S), \forall S \in N$
  - The core can be empty
    - u ***A non-empty core in a superadditive game => stable grand coalition***
- The drawbacks of the core
  - The core is often empty.
  - When the core is non-empty it is often a large set.
  - The allocations that lie in the core are often unfair.



# ***Ex: Cooperative Transmission***

---

- New communication paradigm
  - Exploring broadcast nature of wireless channel
  - Relays can be served as virtual antenna of the source
  - MIMO system
  - Multi-user and multi-route diversity



- Most popular research in current wireless communication
- Industrial standard: IEEE WiMAX 802.16J

# Cooperative Transmission Model

---

- No cooperation (direct transmission), primary user needs power  $P_d$ .
- Cooperative transmission

- **Stage one**: direct transmission. s, source; r, relay; d, destination

$$y_{s,d} = \sqrt{P_0}h_{s,d}x + n_{s,d},$$

$$\text{and } y_{s,r_i} = \sqrt{P_0}h_{s,r_i}x + n_{s,r_i}, \forall i \in \{1, \dots, N\}$$

- **Stage two**: relay retransmission using orthogonal channels, amplified-and-forward

$$y_{r_i,d} = \frac{\sqrt{P_i}}{\sqrt{P_0|h_{s,r_i}|^2 + \sigma^2}}h_{r_i,d}y_{s,r_i} + n_{r_i,d}.$$

- Maximal ration combining at the receiver of backbone node

$$\Gamma = \Gamma_0 + \sum_{i=1}^N \Gamma_i \quad \Gamma_0 = \frac{P_0|h_{s,d}|^2}{\sigma^2} \quad \Gamma_i = \frac{P_0P_i|h_{s,r_i}|^2|h_{r_i,d}|^2}{\sigma^2(P_0|h_{s,r_i}|^2 + P_i|h_{r_i,d}|^2 + \sigma^2)}$$

- To achieve same SNR, power saving for primary user  $P_0 < P_d$



# Main Idea

To get a good position, try to volunteer first



CR users

PR transmission

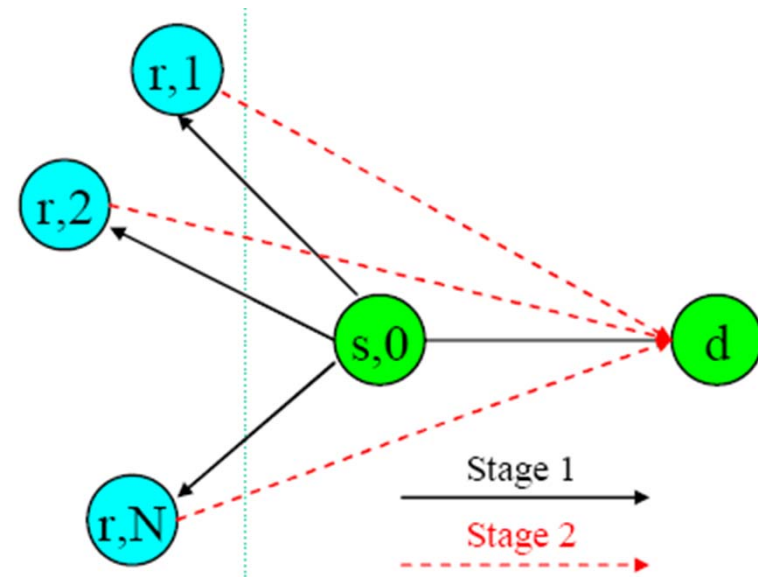


Fig. 2: Coalition Game with Cooperative Transmission

- CR nodes help the PU node reduce transmission power using cooperative transmission, for future rewards of transmission.
- The idea can be formulated by a coalition game.



## ***Other applications of canonical games***

---

- Zhu Han and H. Vincent Poor, "Coalition Games with Cooperative Transmission: A Cure for the Curse of Boundary Nodes in Selfish Packet-Forwarding Wireless Networks", IEEE Transactions on Communications. vol. 57, No. 1, P.P. 203-213, January 2009.
- Rate allocation in a Gaussian multiple access channel (La and Anantharam, 2003)
  - The grand coalition maximizes the channel capacity
  - How to allocate the capacity in a fair way that stabilizes the grand coalition?
    - *The Core, Envy-free fairness (a variation on the Shapley value)*
- Virtual MIMO (W. Saad, Z. Han, M. Debbah, A. Hjørungnes, 2008)
- Allocation of channels in a cognitive radio network when service providers cooperate in a grand coalition (Aram et al., INFOCOM, 2009)
- Any application where
  - The grand coalition forms (no cost for cooperation)
  - Stability and fairness are key issues



# Class II: Coalition Formation Games

---

- Main Properties

- The game is **NOT** superadditive
- Cooperation gains are limited by a cost
  - u *The **grand coalition** is **NOT** guaranteed to form*
- Cluster the network into partitions
- **New issues:** network topology, coalition formation process, environmental changes, etc

- Key Questions

- How can the users form coalitions?
- What is the network structure that will form?
- How can the users adapt to environmental changes such as mobility, the deployment of new users, or others?
- Can we say anything on the stability of the network structure?



## ***Coalition Formation: Merge and Split***

---

- **Merge rule:** merge any group of coalitions where

$$\{\cup_{j=1}^l S_j\} \triangleright \{S_1, \dots, S_l\}$$

- **Split rule:** split any group of coalitions where

$$\{S_1, \dots, S_l\} \triangleright \{\cup_{j=1}^l S_j\}$$

- A decision to merge (split) is an agreement between all players to form (break) a new coalition
  - Socialist (social well fare improved by the decision)
  - Capitalist (individual benefit improved)



# ***Merge and Split: Properties***

---

- Any merge and split iteration converges and results in a final partition.
- Merge and split decision
  - Individual decision
  - Coalition decision
  - Can be implemented in a distributed manner with no reliance on any centralized entity
- Using the Pareto order ensures that no player is worse off through merge or split
  - Other orders or preference relations can be used



# *Stability Notions*

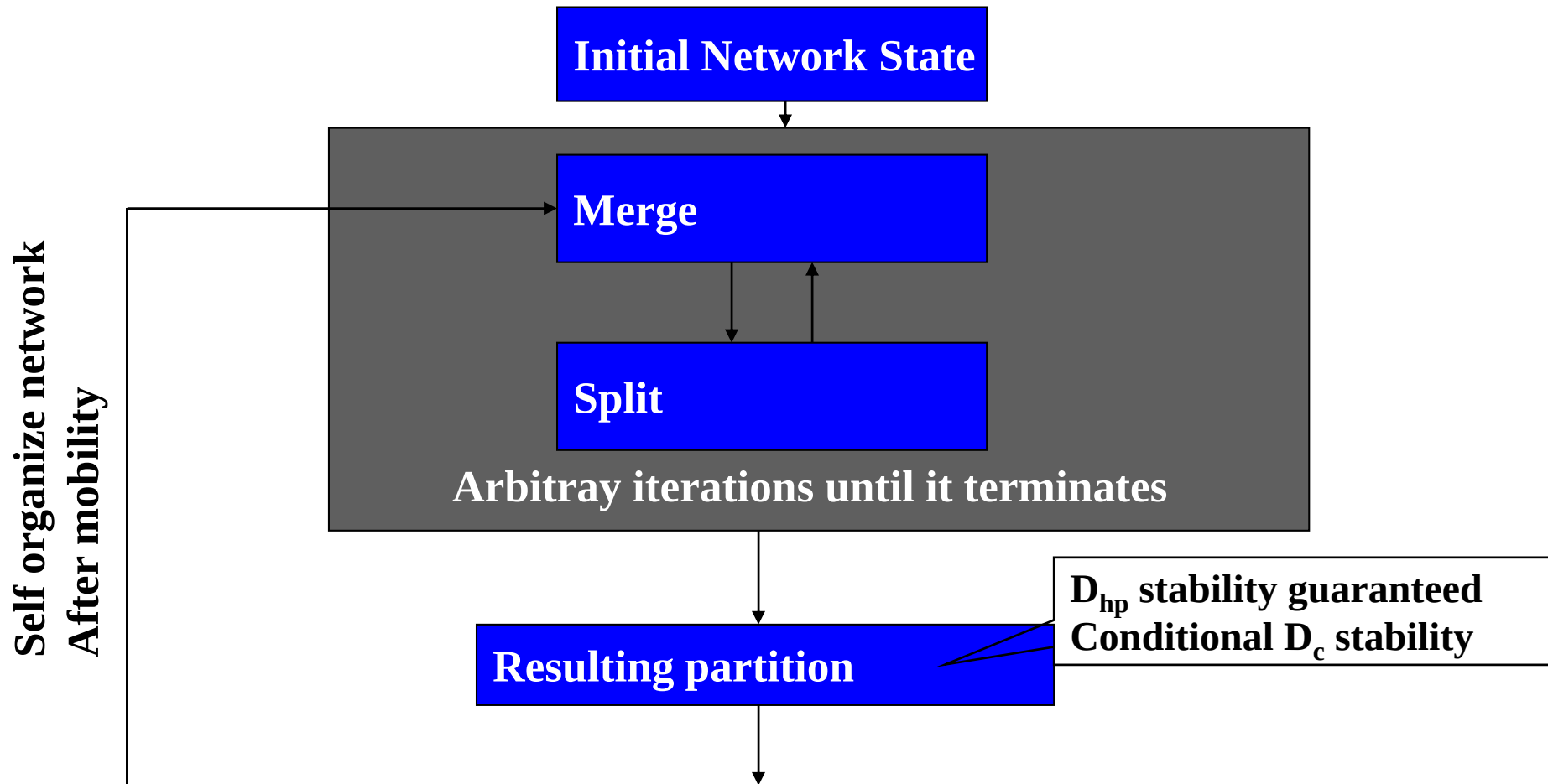
---

- $D_{hp}$  stable
  - No users can defect via merge/split
  - Partition resulting from merge and split is  $D_{hp}$  stable
- $D_c$  stable
  - No users can defect to form a new collection in  $N$
  - A  $D_c$  stable partition is **socially optimal**
  - When it exists, it is the **unique** outcome of any merge and split iteration
  - Strongest type of stability

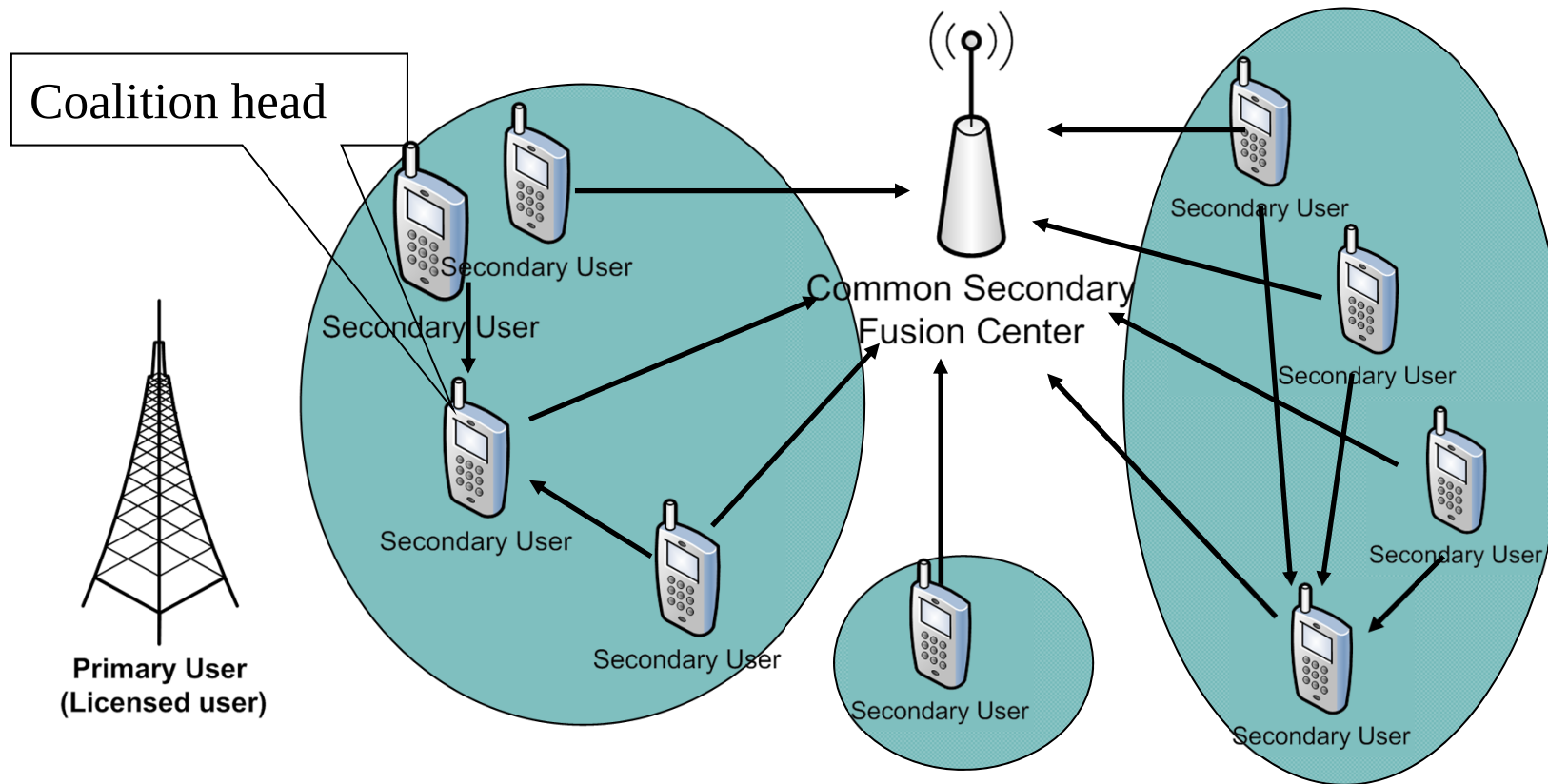


# *Merge and Split algorithm*

---



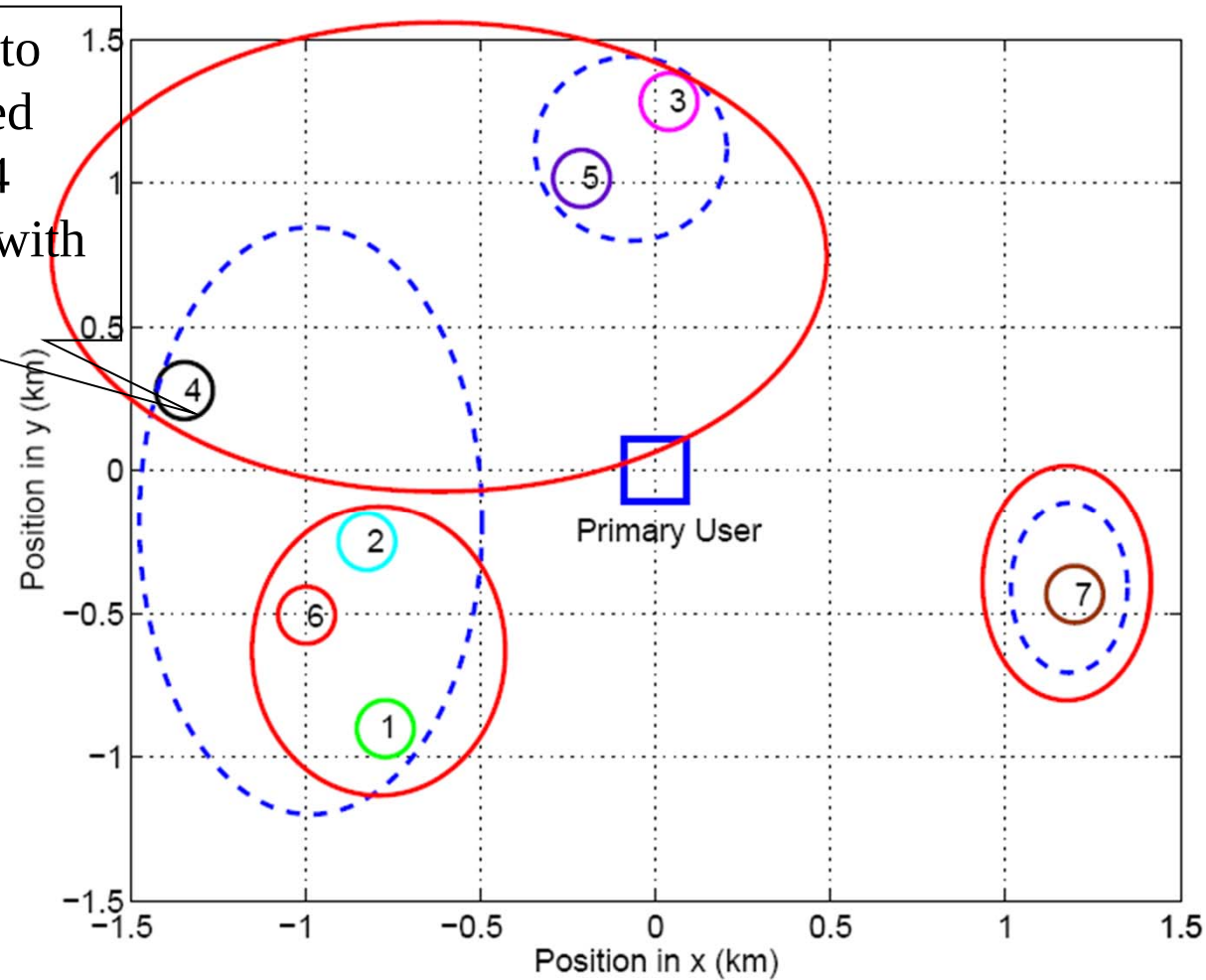
# ***Distributed Collaborative Sensing***



- Distributed collaborative sensing between the users with **no centralized** fusion center
- Which groups will form?
  - **Coalitional games!**

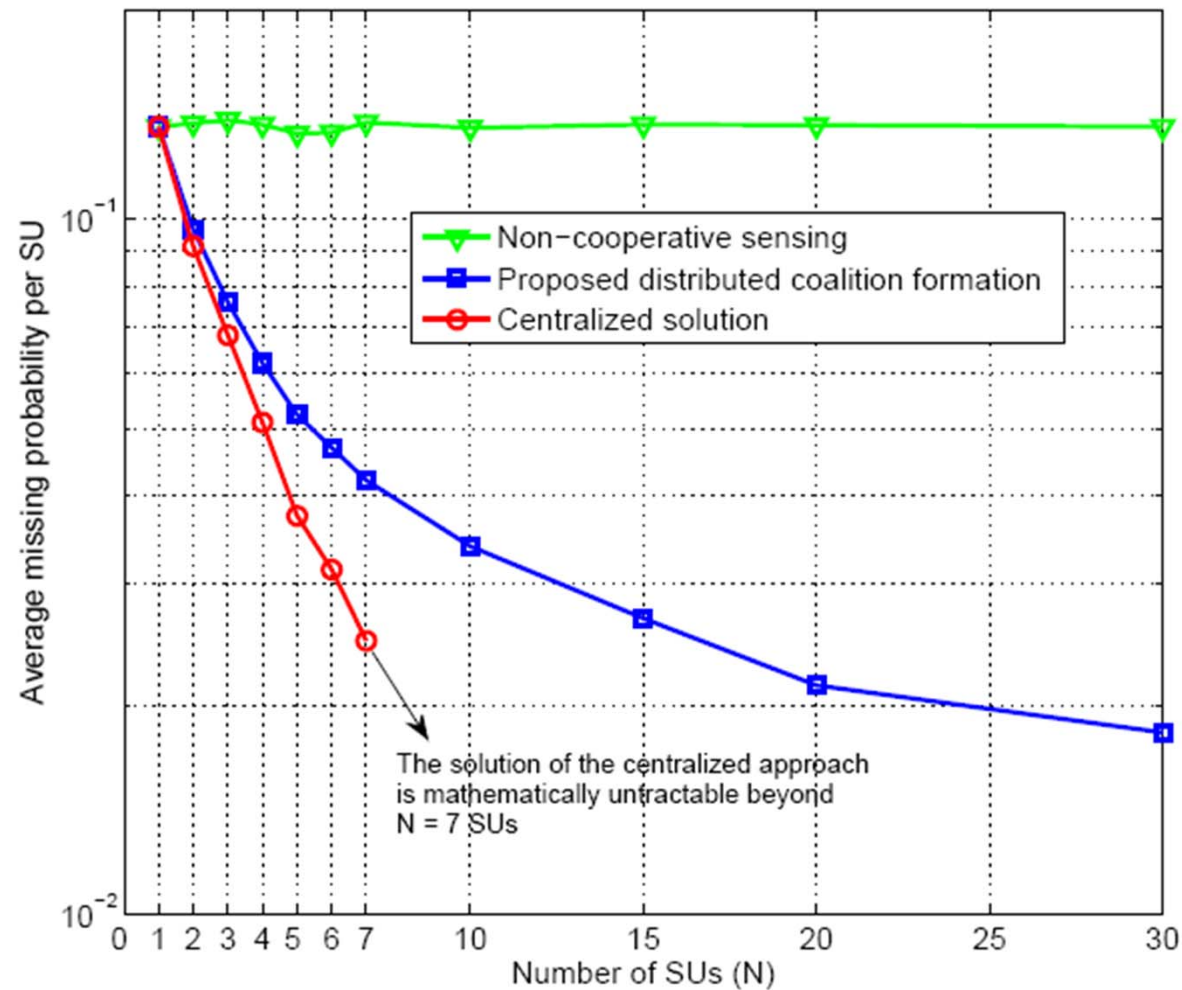
# Simulation Results

When allowed to make distributed decisions, SU 4 prefers to stay with {2,1,6}

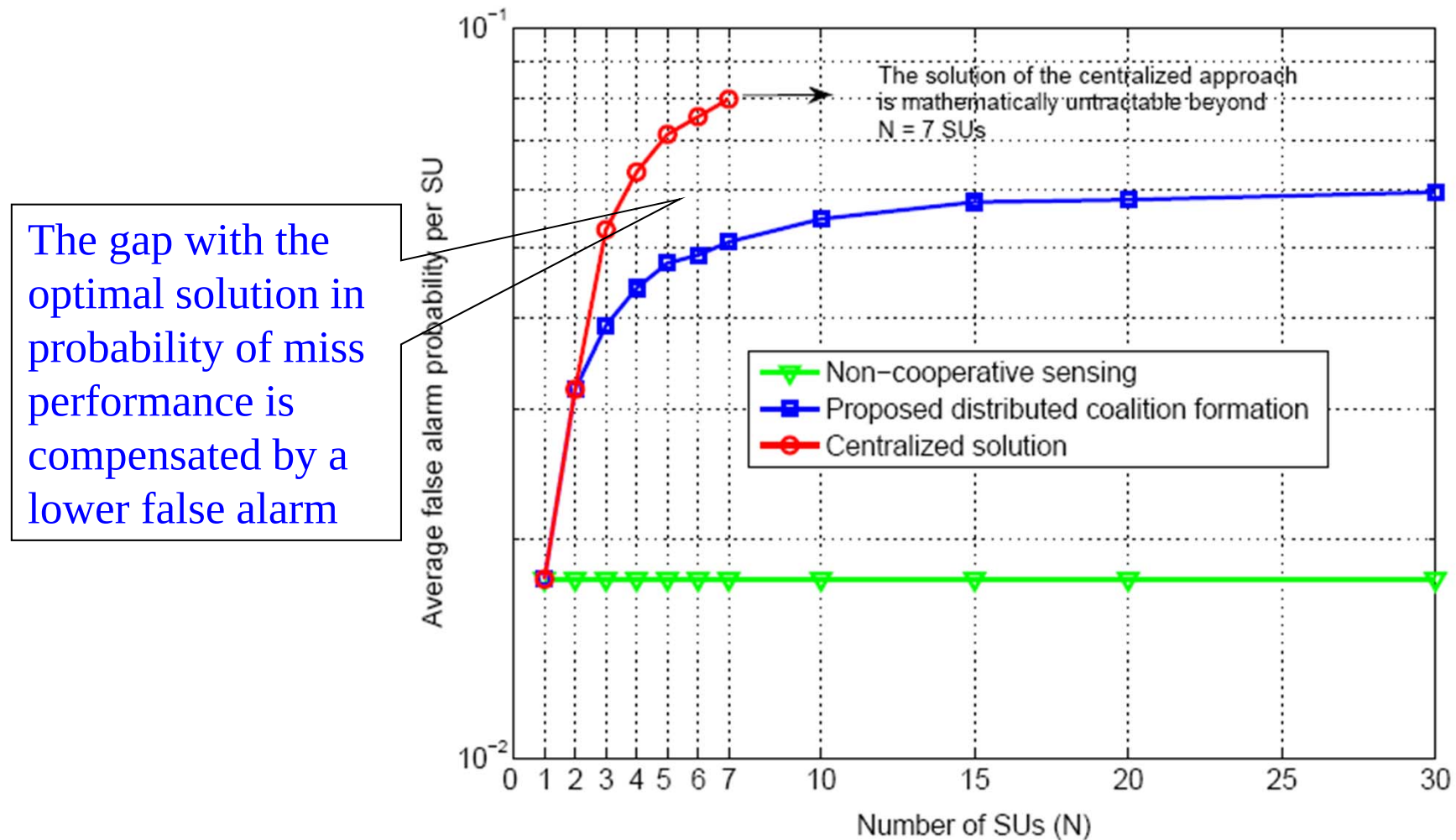




# Simulation Results (1)



# Simulation Results (2)



# ***Other applications of coalition formation***

---

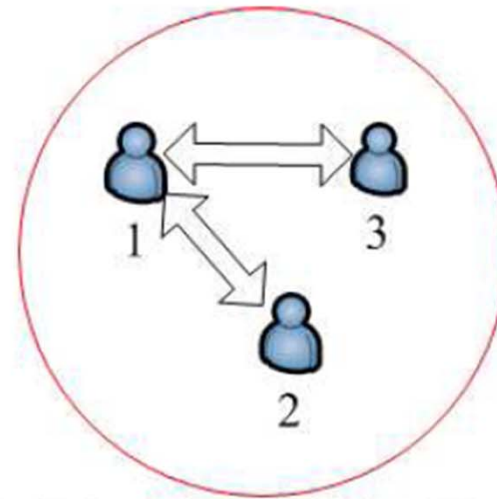
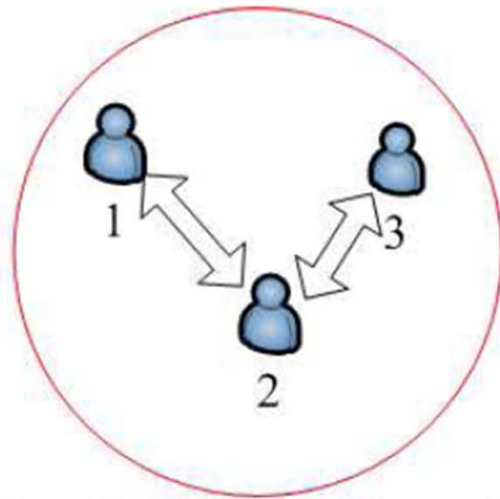
- Coalitional games for topology design in wireless networks
  - **Physical layer security**
    - u *Merge-and-split for improving secrecy capacity*
    - u **W. Saad, Z. Han, T. Basar, M. Debbah and A. Hjørungnes, “Physical layer security: coalitional games for distributed cooperation,” WiOpt, 2009**
  - **Task allocation among UAVs in wireless networks**
    - u *Hedonic coalition formation*
    - u **W. Saad, Z. Han, T. Basar, M. Debbah and A. Hjørungnes, “A selfish approach to coalition formation in wireless networks,” GameNets, 2009**
  - **Vehicular Network**
    - u *“Coalition Formation Games for Distributed Roadside Units Cooperation in Vehicular Networks”, JSAC Jan. 2011*
  - **Endless possibilities**
    - u *Study of cooperation when there is cooperation with cost*
    - u *Topology design in wireless networks*
    - u *Beyond wireless: smart grid*



# Class III: Coalition Graph Games

---

- Main properties
  - The game is in **graph form**
    - *May depend on externalities also*
  - There is a **graph** that connects the players of every coalition
  - Cooperation with or without cost
  - **A Hybrid type of games:** concepts from classes I and II, as well as non-cooperative games



# Coalition Graph Games

---

- First thought of by Myerson, 1977, called “Coalitional games with communication structure”
  - Axiomatic approach to find a Shapley-like value for a coalitional game with an underlying graph structure
  - Coalition value depends on the graph
  - The dependence is only based on **connections**
- Key Questions
  - How can the users form the **graph** structure that will result in the network?
  - If all players form a single graph (grand coalition with a graph), can it be stabilized?
  - How can the users adapt to environmental changes such as mobility, the deployment of new users, or others?
  - What is the effect of the graph on the game?



# ***Applications of Coalitional Graph Games***

---

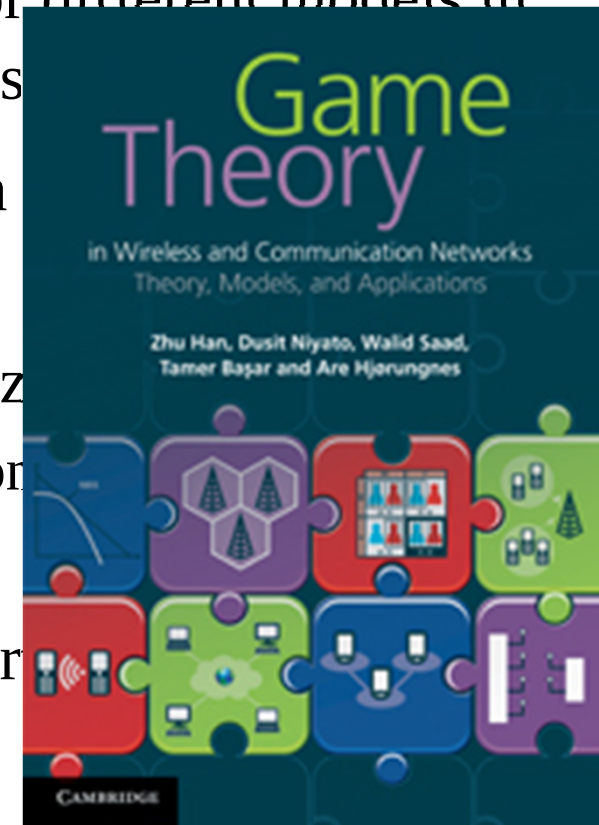
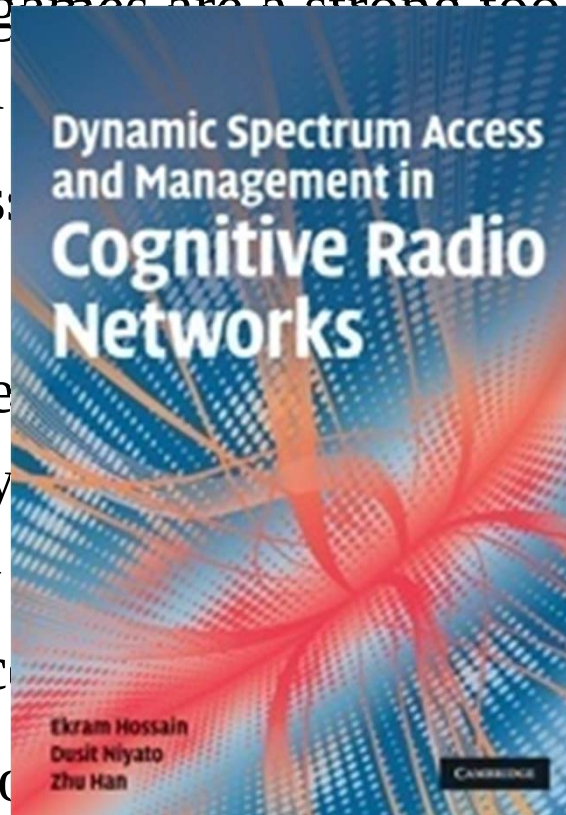
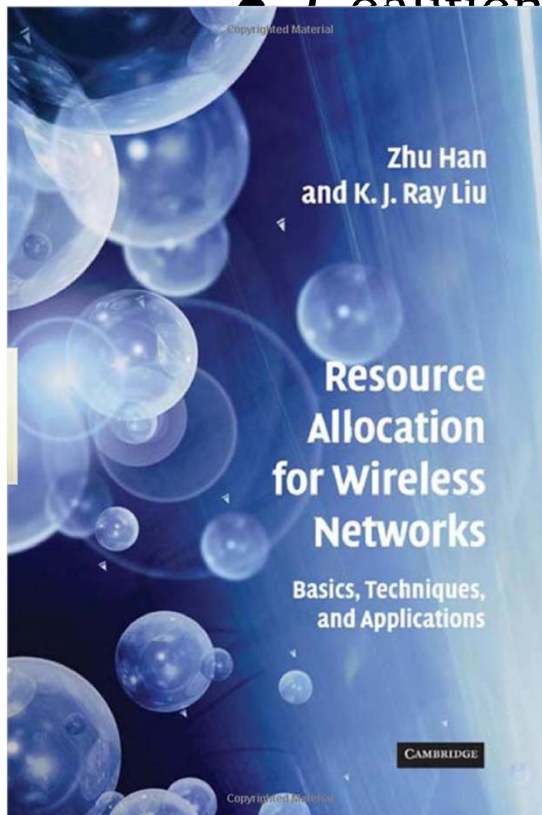
- Coalitional graph games for network formation
  - **WiMAX IEEE 802.16j/LTE**
    - u *Network formation game for uplink tree structure formation*
    - u **W. Saad, Z. Han, M. Debbah, and A. Hjørungnes,**  
***“Network formation games for distributed uplink tree construction in IEEE 802.16j,” in proc. GLOBECOM 2008***
    - u **W. Saad, Z. Han, M. Debbah, A. Hjørungnes, and T. Basar,**  
***“A game-based self-organizing uplink tree for VoIP services in IEEE 802.16j,” ICC 2009***
  - **Routing in communication networks**
    - u *See the work by Johari (Stanford)*
  - **Many future possibilities**
    - u *The formation of graphs is ubiquitous in the context of communication networks*





# Summary of coalitional game

- Cognitive radio network and its basic problems
- Coalitional games are a strong tool for different models in



*Questions?*

---

Thank you very much

